How To Be A Better Babylon Player

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Rules of the Game

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- Two players take turns combining piles of tokens
- Piles can be combined if they have the same color token on **top** or the same **height**.
- A player wins if they are the last player to combine a pile.

Sample Game

On-line implementation of Babylon

Game Tree Analysis

- Who will win Babylon?

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- Create game tree computationally

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- Who will win Babylon?
- Create game tree computationally
- Explore 600+ game states
- Second player if played perfectly

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- All moves known to all players.

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- Example of N: one red and one blue
- Example of P: three blues

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- The game as designed: $(4, 12, \{3, 3, 3, 3\})$
- Our focus: $(2, m, \{p, q\})$ where $p + q = m$
- Convention: red tokens least common, blue tokens most common

How Many Moves?

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- Game ends with either one or two piles
- Game lasts either 3 or 4 turns
- If I move first, I want the game to last how long?

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- Winning move: cover the red token

Two Red Tokens With Even Number of Tokens

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- If I start, how many piles do I want?
- If $2m = 4$, cover a red with a red
- If $2m > 4$, cover a red with a blue
  - If opponent created a pile of size two with red on top, cover it
  - If not, cover the last red token

Table of Possible Games

Blue

		1	2	3	4	5	6	7	8
	1	N	N	N	N	N	N	N	N
Red	2		N	N	N	N	N	N	N
	3			P	N	P	N	P	N
	4				P	N	P	N	P
	5					P	N	P	N
	6						P	N	P
	7							P	N
	8								P

Empirical investigation with Java programming
Distributed computing through Condor

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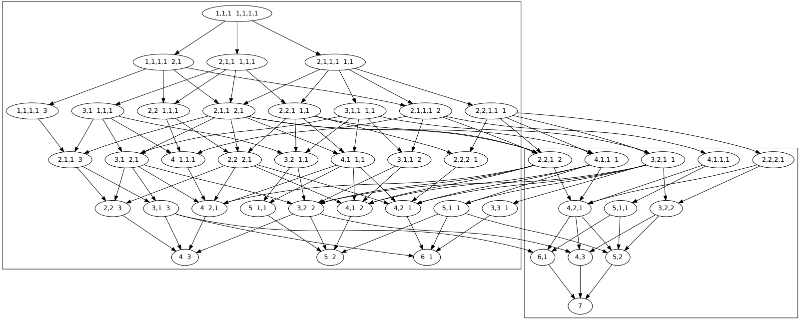
- Midgame position $(3, 2 : 1, 1)$

Game Tree for $(2, 7, \{3, 4\})$



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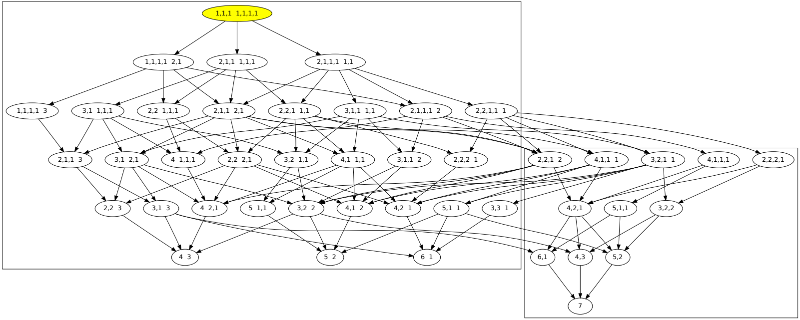

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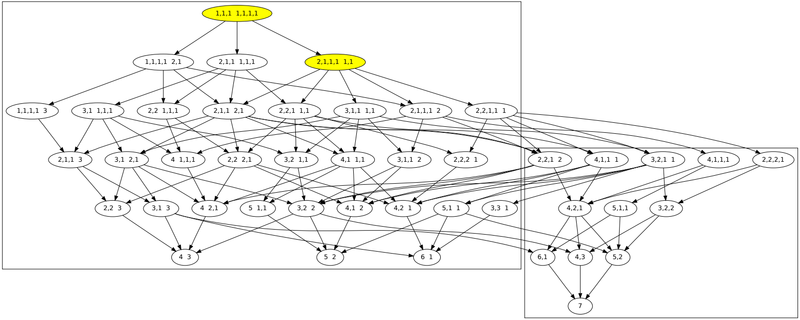

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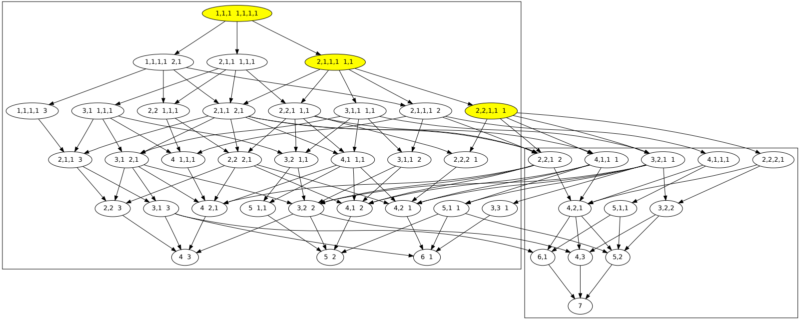

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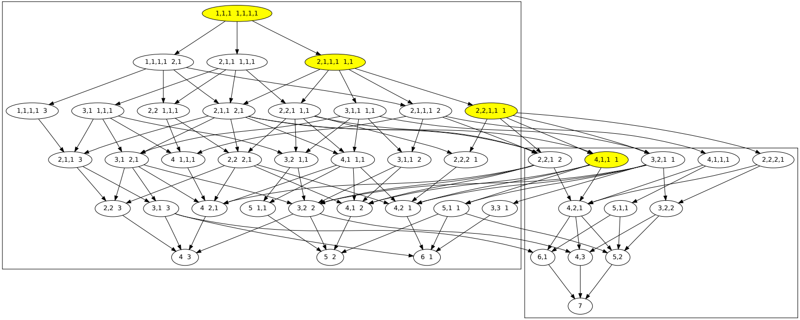

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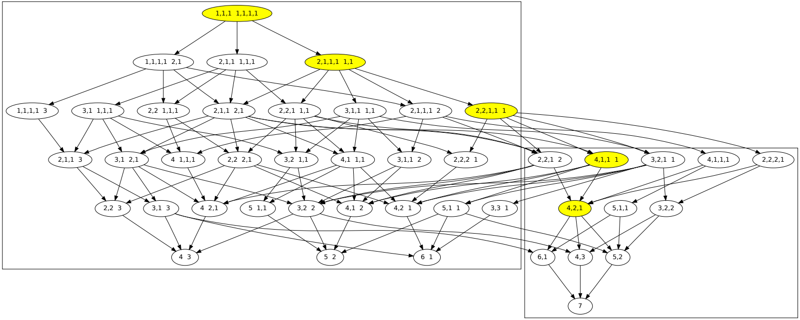

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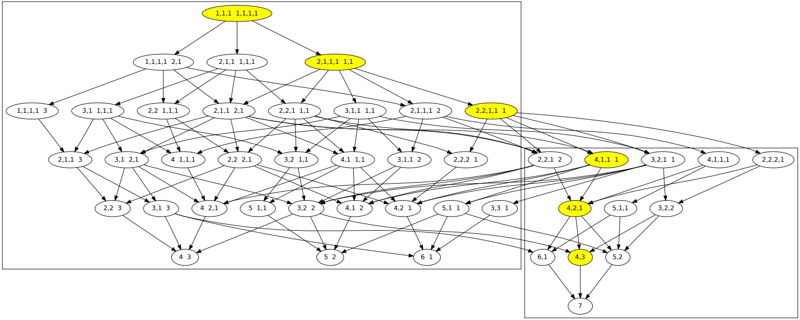

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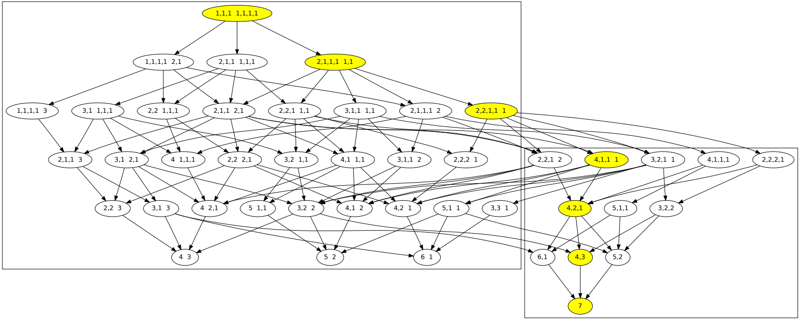

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- How many moves do we want?
- We want two piles with different colors on top.

Some Important Definitions

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  - only one pile of one color
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- Our strategy: four-stage process

Stage 1

Place a red token on a blue token

Stage 2

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- We are now PPD!

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- Otherwise, stay in Stage 3

Stage 4: SMP

Make any move that leaves the game in an SMP position

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- If majority piles have different sizes, a move exists
- If all majority piles have the same size...
- ... then we have an even number of tokens $\rightarrow\leftarrow$

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- even number of tokens, two red - N
- odd number of tokens - N

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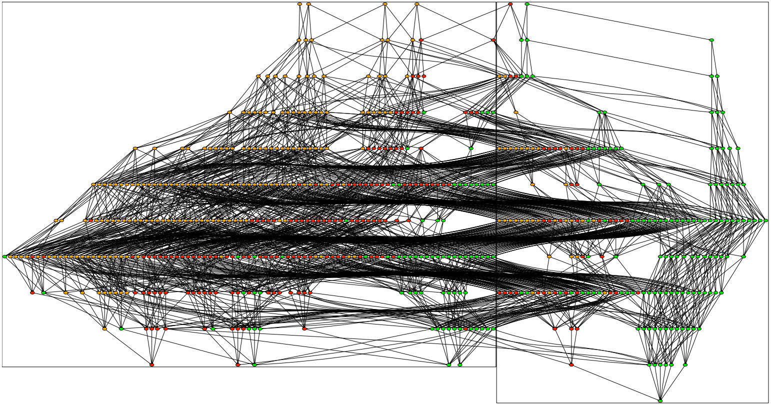

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- What about more colors?